Phyx 320
Modern Physics

January 25, 2021

Reading: 36.5 - 36.8

Homework #1 and Reading Reflection Due Tuesday 11:59 pm

Constant Speed of Light

All laws of physics are independent of reference frame.

This implies that the speed of light is independent of reference frame

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \ \frac{m}{s}$$

We have to change our assumptions about space and time

Independent of reference frame

$$c = \frac{dx}{dt}$$

Space and time must be frame dependent

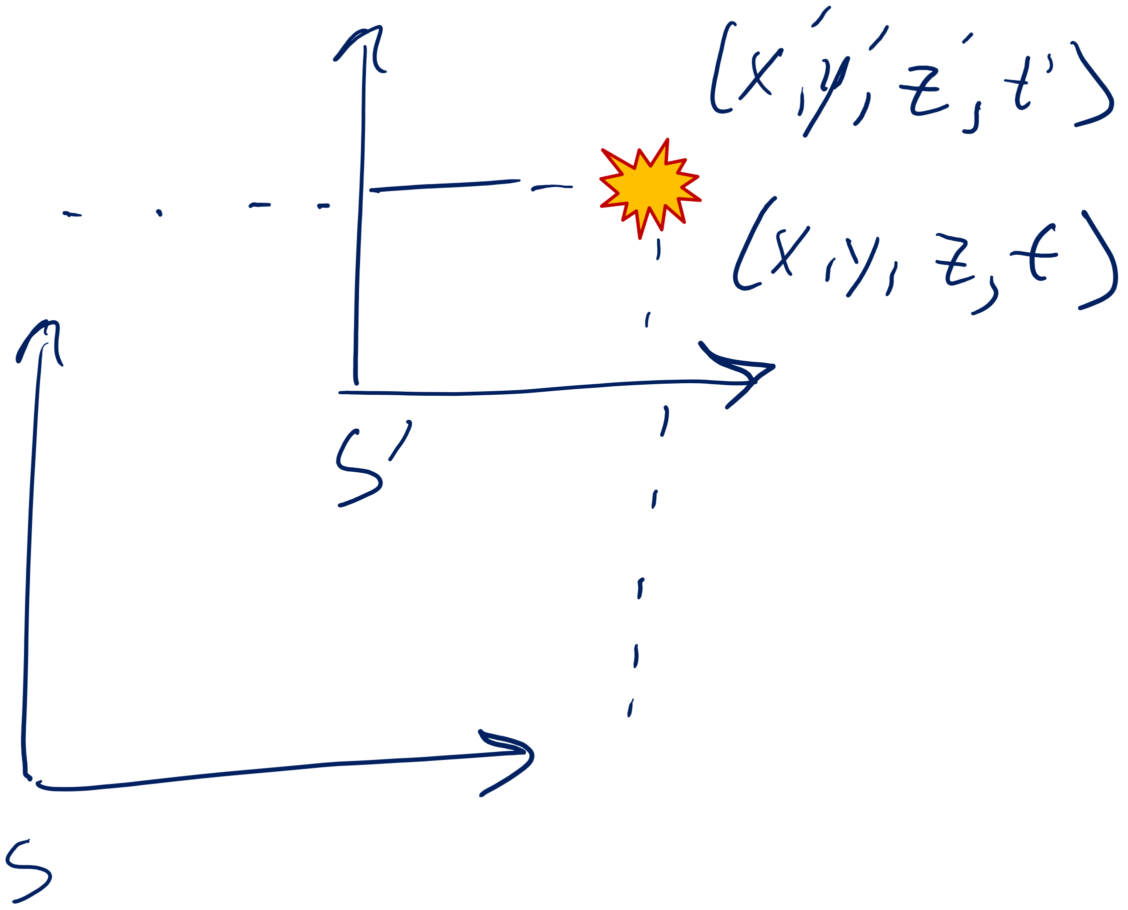
Spacetime Events

We need to be clear about what we mean when we talk about space and time

Both space and time are implicitly measured by events

- Length of an object is measured by the emission of light from it's two ends
- Time is measured by the progression of events

Since both space and time are frame dependent, we now have primed time coordinates

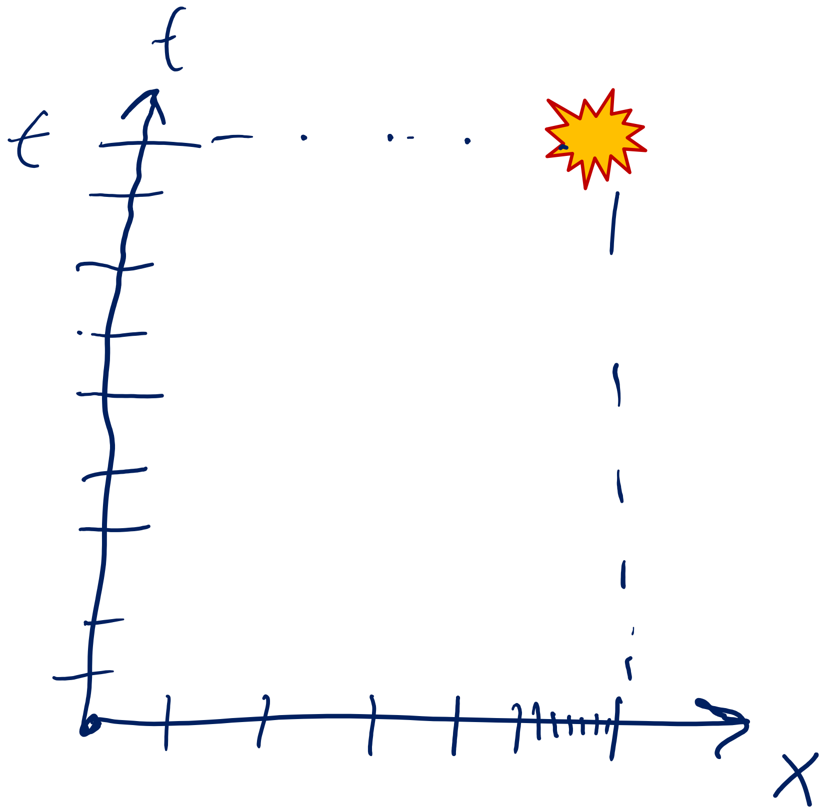


Spacetime Measurements

Just like space we need to set up a "grid" for time, like frames of a video

Grid spacing is infinitesimal, time is a continuum not discrete (just like space)

The rate that time flows will depend on frame



Simultaneity

Since speed of light is finite, information takes time to flow from event to observer

- Information can be slower (sound) but can't be faster than light

Observed spacetime coordinate is **not** the same thing as when the event happened

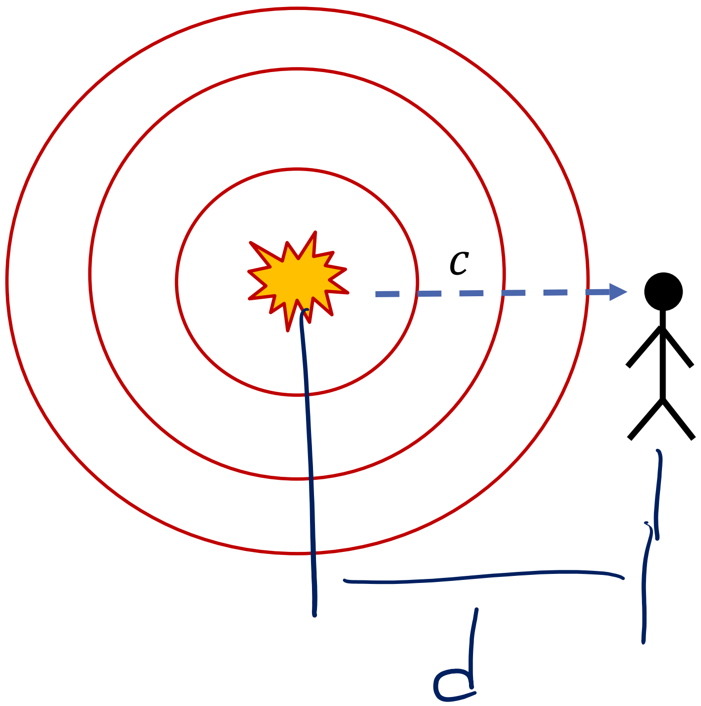
As you look further away you see farther back in time

- Sun = 8 min 11.27 s ago
- Other side of Milky Way = 105,700 years ago (earliest human-made structures)

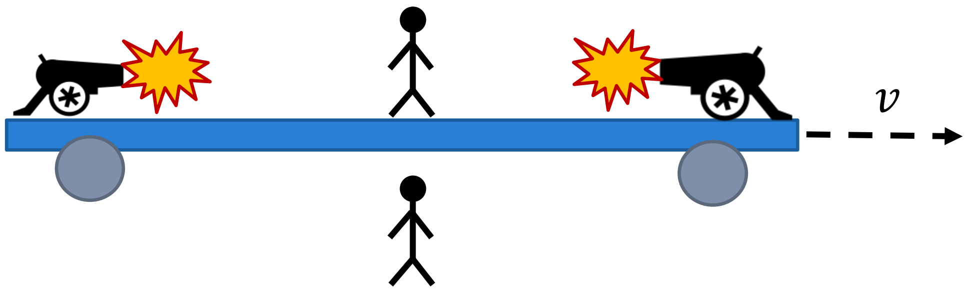
Nice way to measure astronomical distances

- Light-year = distance light travels in one Earth year (ly

$$t_{obs} = \frac{d}{c} + t_{event}$$



Simultaneity Paradox



Let’s say we have a cart traveling close to the speed of light

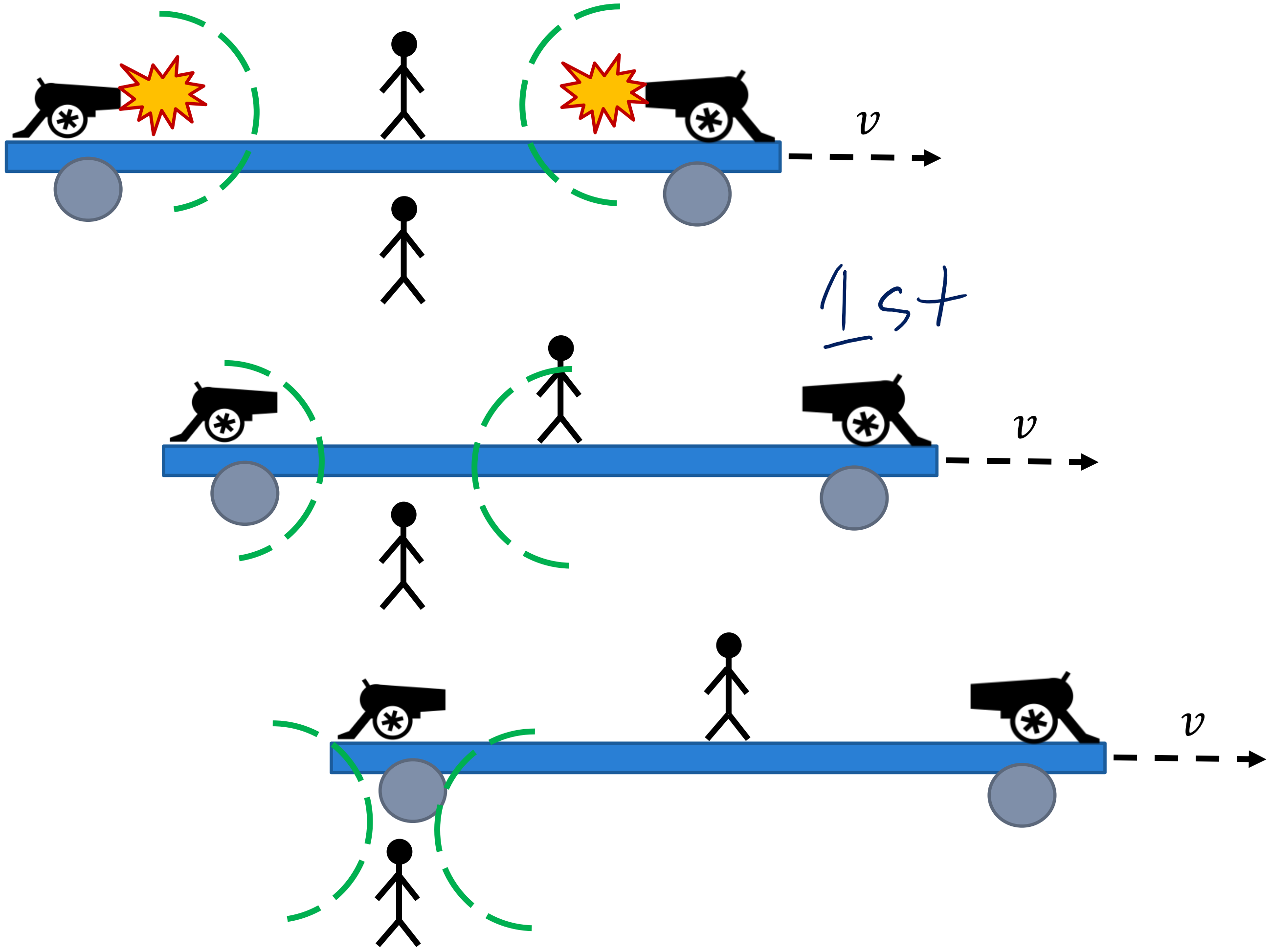
Two cannons fire and each person must choose which cannon the person on the cart saw light from first

Simultaneity Paradox

From the ground's perspective both cannons fire at the same time

Equal distance from the person on the ground

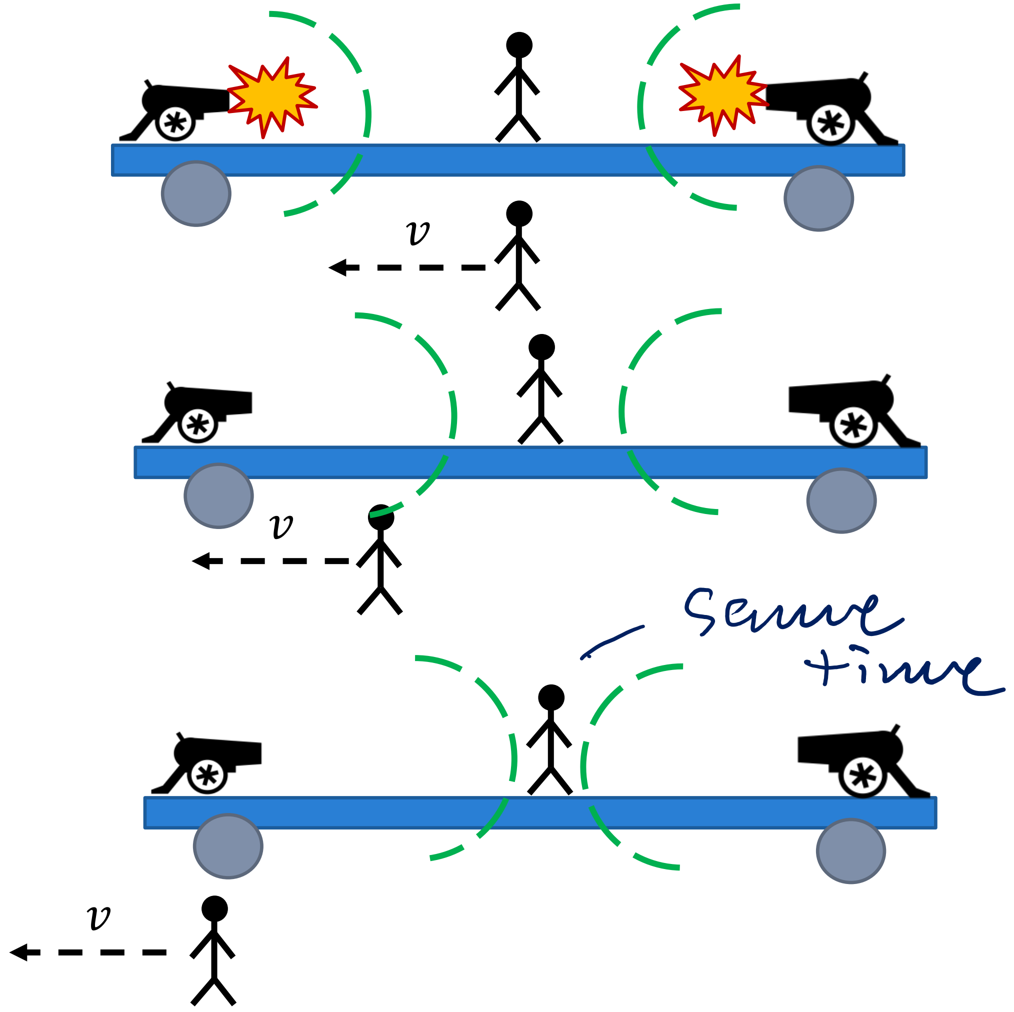
Person on the ground concludes person on the train must see light from right cannon first



Simultaneity Paradox

From cart's perspective if both cannon's fire simultaneously then light waves show up at same time

How can this be?

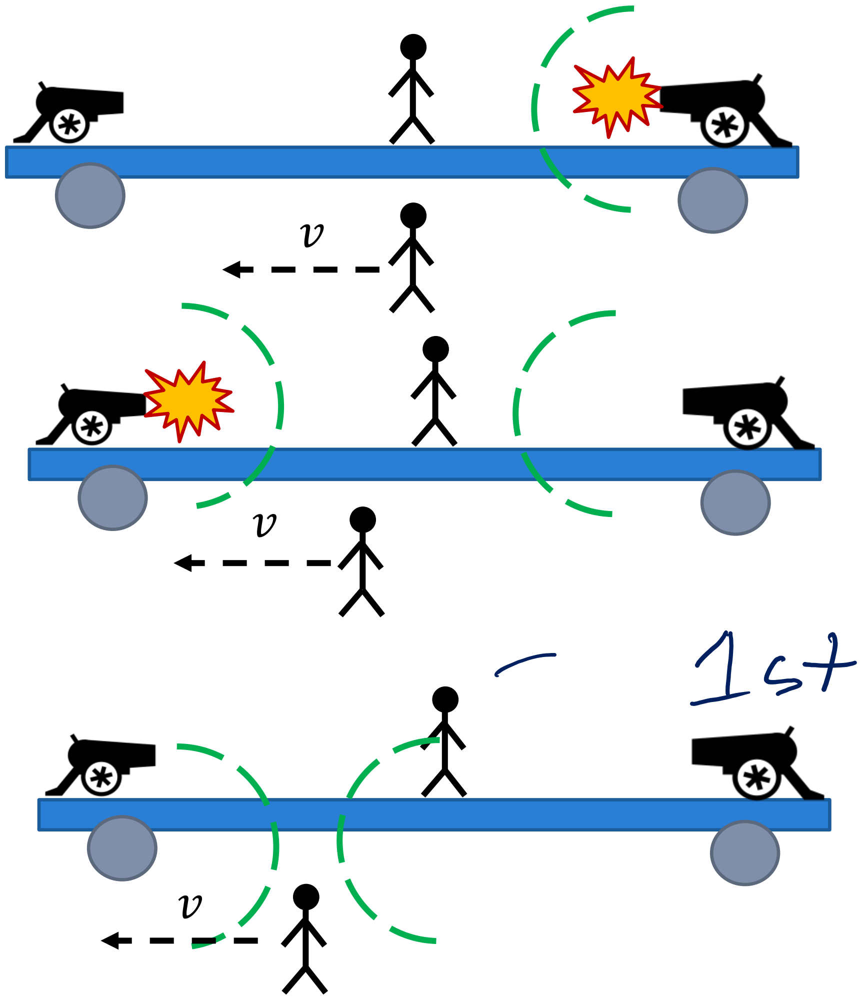


Simultaneity Paradox

Assumed cannons fire at the same time in cart's reference frame

But to keep the conclusions consistent then right cannon had to fire first

Simultaneous events in one frame are not guaranteed to be simultaneous in another

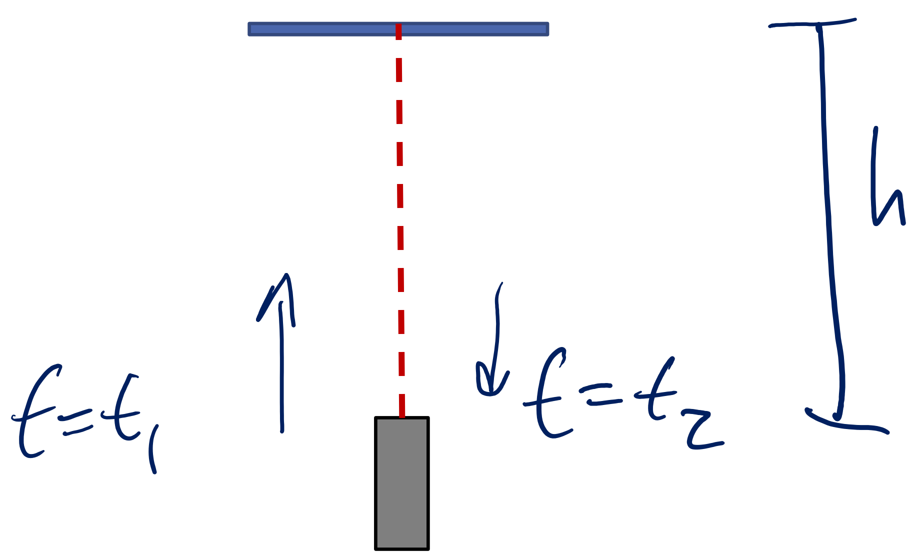


Time Dilation

How does time appear to flow in different reference frames?

Let's make a clock with light

- Shoot pulse of light off mirror at $t = t_1$
- Measure time it comes back $t = t_2$



$$2h = c\Delta t$$

$$h = \frac{c\Delta t}{2}$$ ← laser rangefinder

$$\Delta t = t_2 - t_1$$

Time Dilation

Now let's put this on a cart traveling at some velocity = v

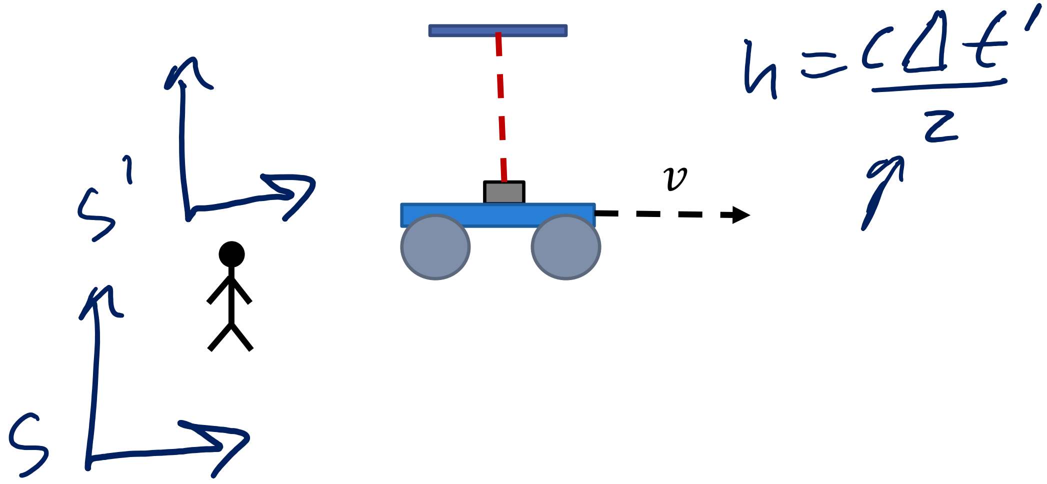
$$h^2 + \left(\frac{v\Delta t}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2$$

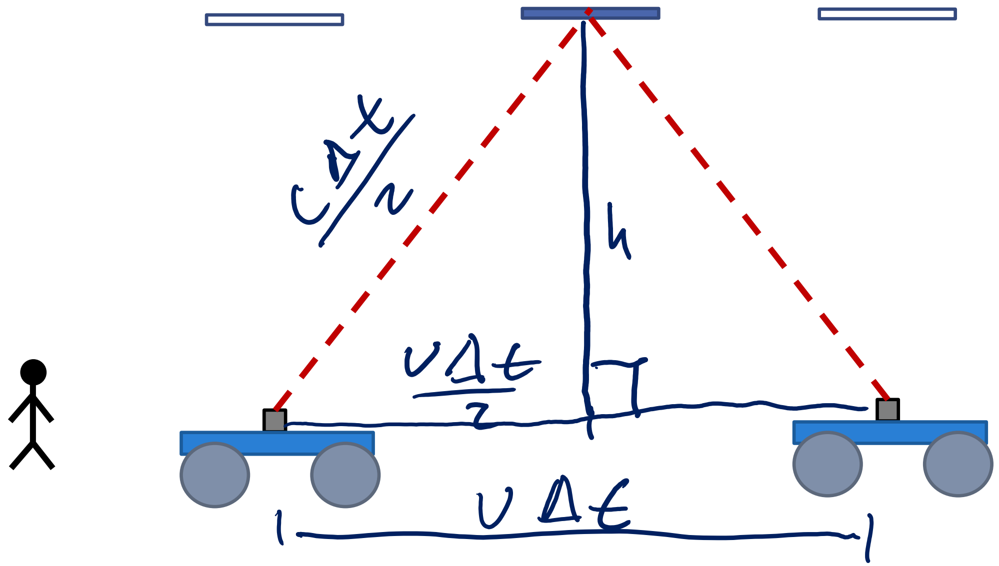
$$\left(\frac{c\Delta t'}{2}\right)^2 + \left(\frac{v\Delta t}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2$$

$$c^2\Delta t'^2 = (c^2 - v^2)\Delta t^2$$

$$\Delta t = \sqrt{\frac{c^2}{c^2 - v^2}}\,\Delta t'$$

$$\Delta t = \sqrt{\frac{1}{1 - (v/c)^2}}\,\Delta t'$$





Proper Time

$$\Delta\tau = \Delta t'$$

$$\Delta t = \frac{1}{\sqrt{1-(v/c)^2}}\,\Delta\tau \qquad \beta = v/c$$

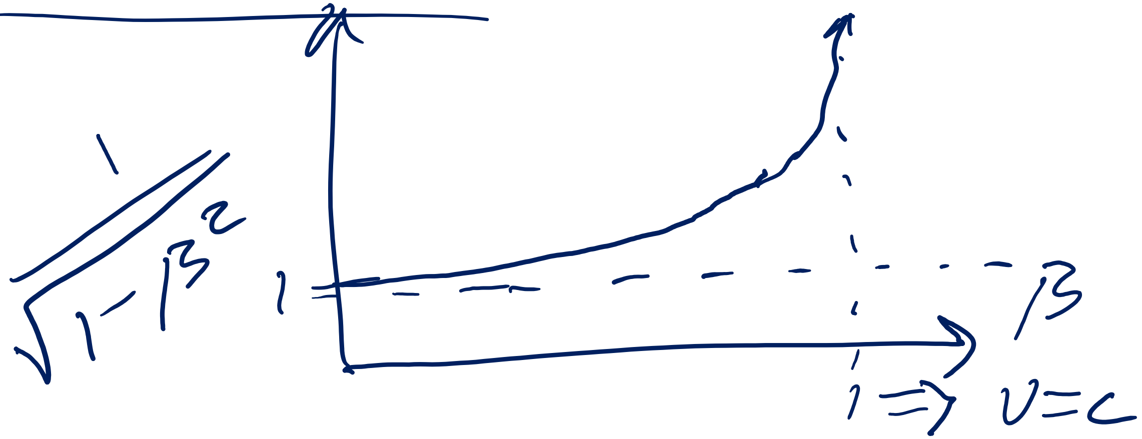
$$\Delta t \geq \Delta\tau$$

Time interval measured in rest frame of clock gets special name: **Proper Time**

Defines a unique reference frame for that clock

The time interval between two clock ticks is always shortest in the rest frame of the clock

Moving clocks run slower

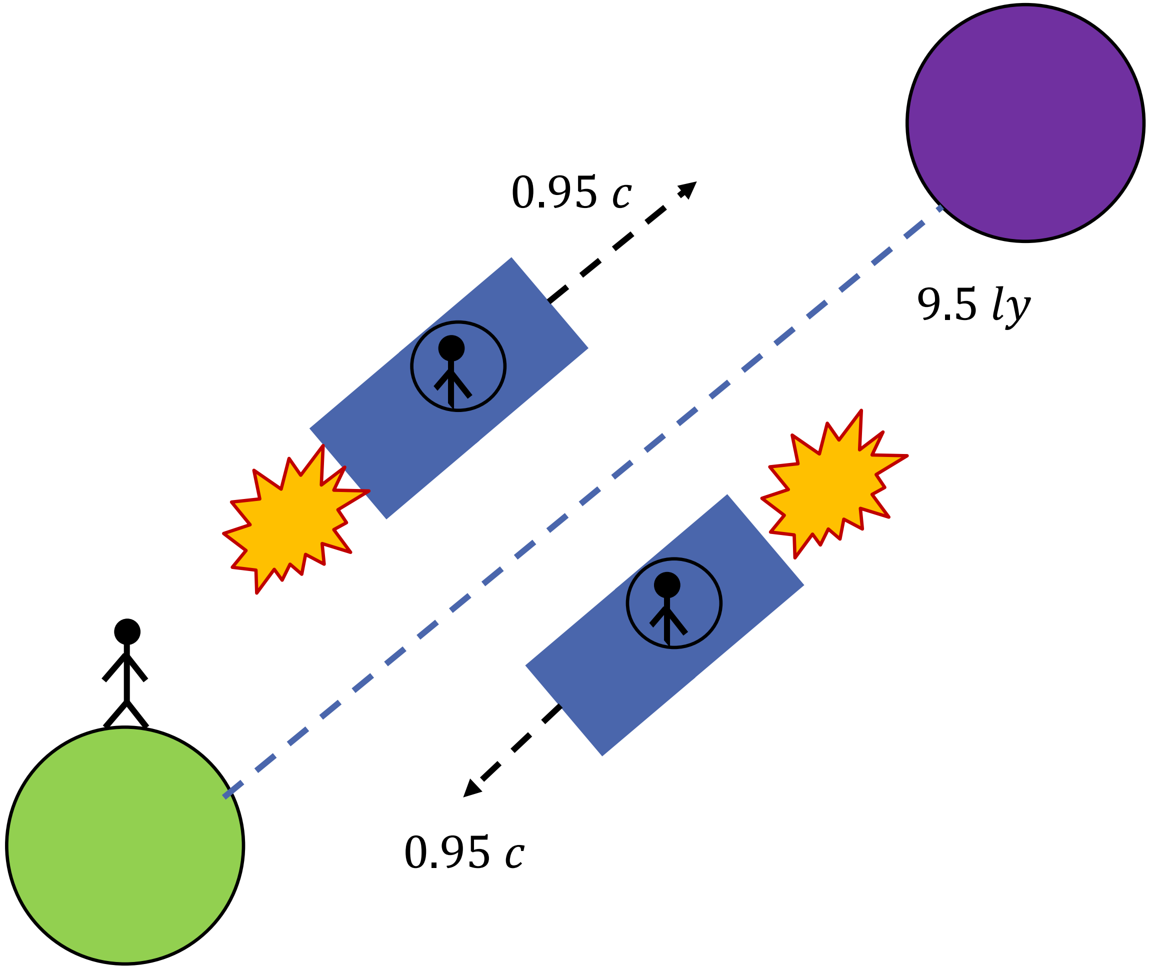


Twin Paradox

Two twins that are 25 years old

One launches out to a star 9.5 light years away traveling at 0.95c

Promptly turns around and comes back to earth



Twin Paradox

Clocks on starship run slower

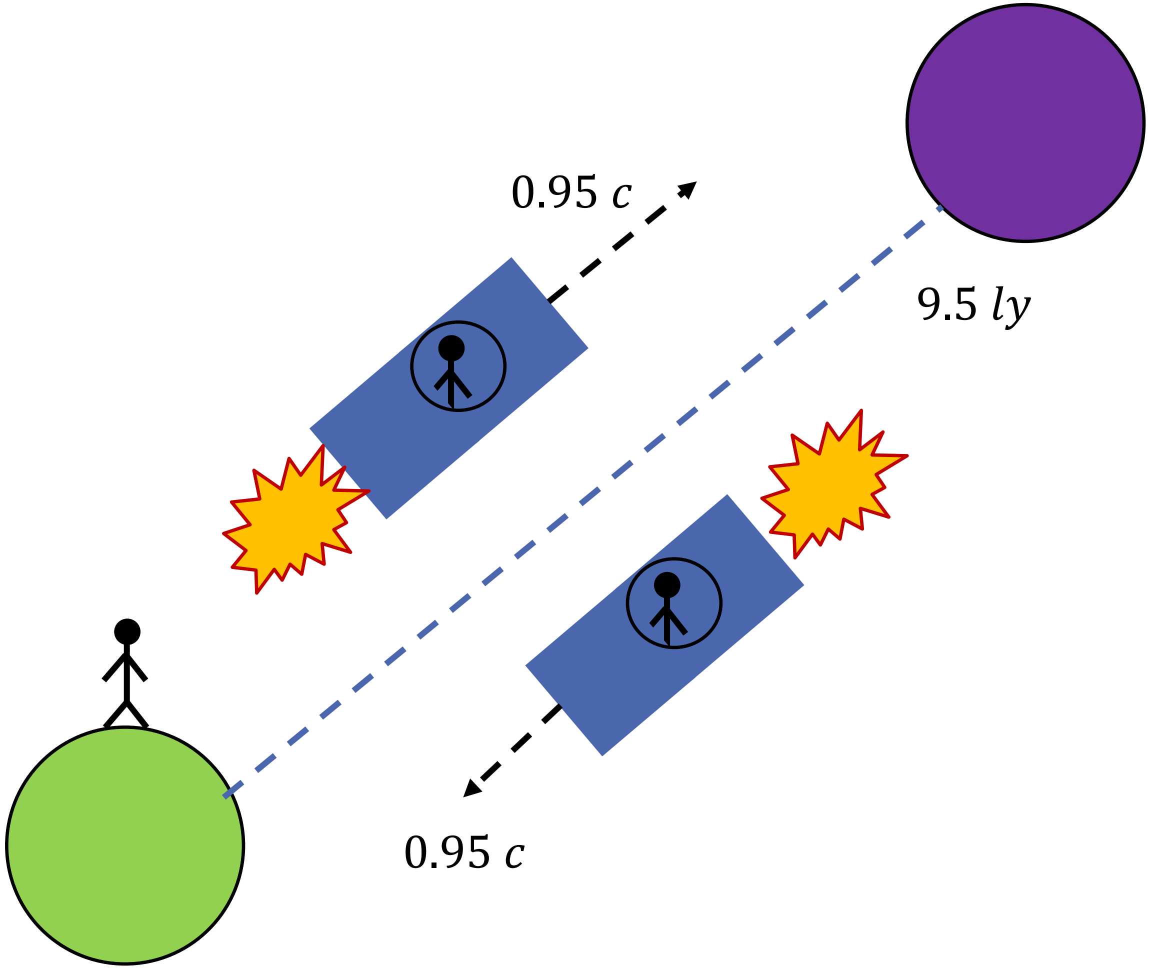
- Including human heart

Person on earth calculates age of person on starship

$$\Delta t_E = \frac{2(9.5)\,ly}{0.95c} = 20 \text{ years}$$

$$\Delta t_{space} = \frac{1}{\sqrt{1-\beta^2}} \Delta t_E$$

$$= 6.25 \text{ years}$$



Twin Paradox

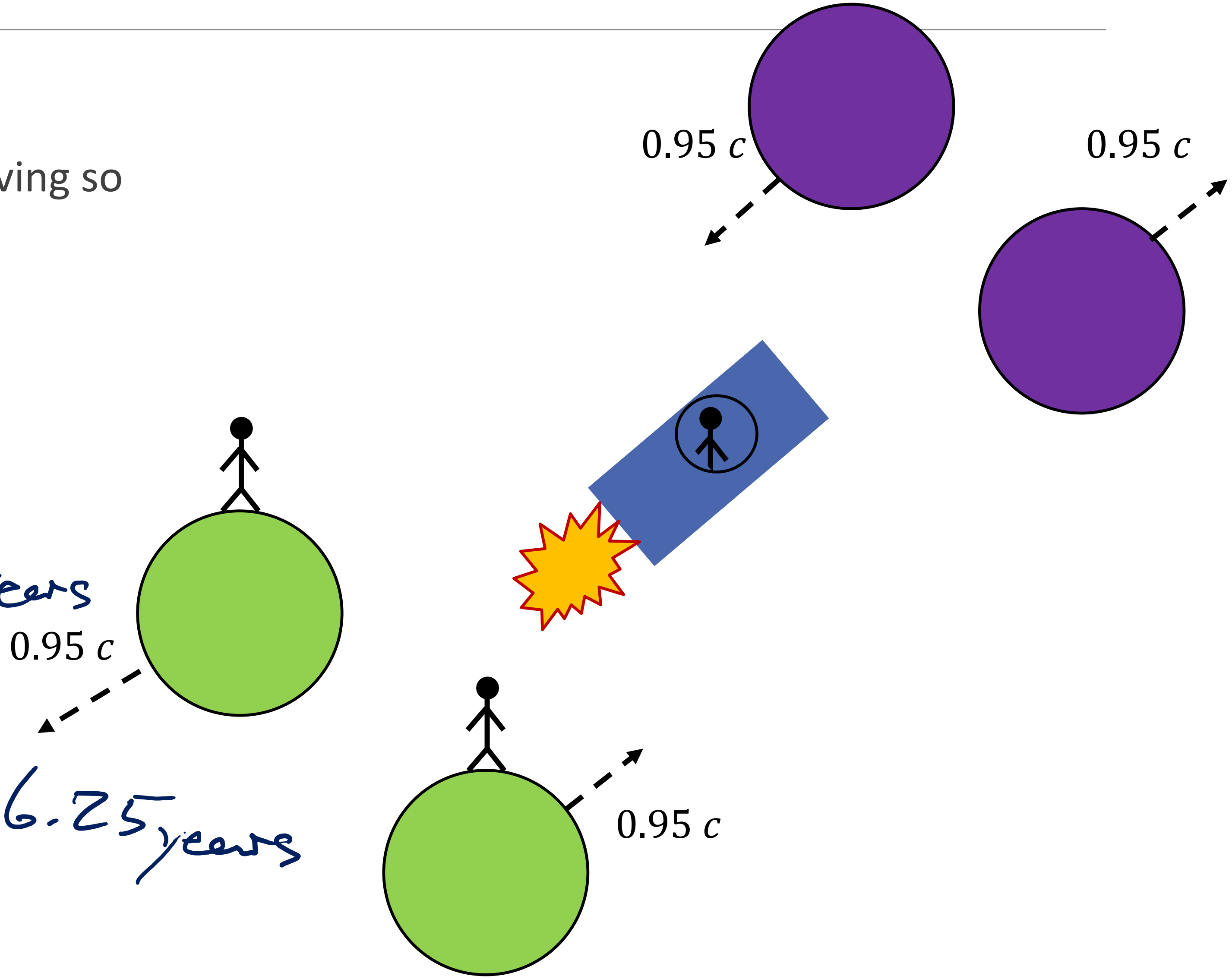
From starship perspective earth is moving so person on starship can do the same calculations

Concludes twin on earth is younger!

Who is actually younger?

$$\Delta t_{space} = \frac{2(9.5\,ly)}{0.95c} = 20 \text{ years}$$

$$\Delta t_E = \frac{1}{\sqrt{1-\beta^2}}\Delta t_{space} = 6.25 \text{ years}$$



Twin Paradox

Calculations from person on earth are correct

Person in starship is in a non-inertial reference frame

- When ship accelerates special relativity doesn't apply!

Twin in starship is younger than twin on earth

Scott Kelly spent 520 days on the International Space Station traveling 28,160 km/h, gained 15 ms on his twin Mark Kelly